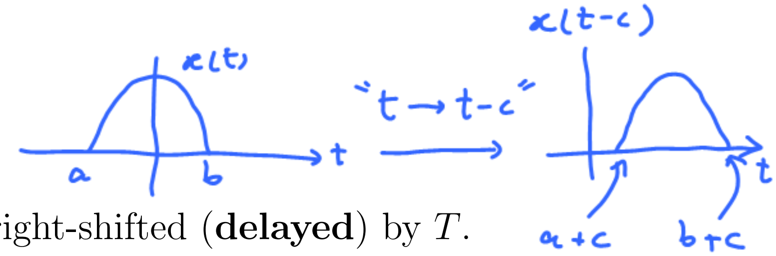
B Time Manipulation

B.1. Consider a function of time $x(t)$.



(a) **Time shifting**:

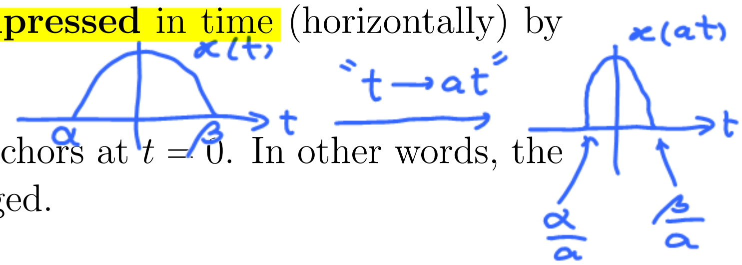
(i) When $T > 0$, $x(t - T)$ is $x(t)$ right-shifted (**delayed**) by T.

(ii) When $T < 0$, $x(t - T)$ is $x(t)$ left-shifted (**advanced**) by T.

(b) **Time scaling** (horizontal scaling):

(i) When $0 < a < 1$, $x(at)$ is $x(t)$ **expanded** in time (horizontally) by a factor of $\frac{1}{a}$.

(ii) When $a > 1$, $x(at)$ is $x(t)$ **compressed** in time (horizontally) by a factor of a.



- Note that the signal remains anchors at $t = 0$. In other words, the signal at $t = 0$ remains unchanged.

(c) **Time inversion** (or folding):

- $x(-t)$ is the mirror image of $x(t)$ about the vertical axis.

Example B.2. A function of the form $x\,(mt + c)$ can be viewed as

(a) $x\,((mt) - (-c))$: First right-shift $x(t)$ by $-c$. (Equivalently, left-shift $x(t)$ by c.) Then scale horizontally by a factor of $\frac{1}{m}$.

(b) $x\left(m\left(t - \left(-\frac{c}{m}\right)\right)\right)$: First scale $x(t)$ horizontally by a factor of $\frac{1}{m}$. Then, right-shift by $-\frac{c}{m}$.

These two approaches are illustrated in Figure 89.

Alternatively, it may be easier to look at where the key points in the plot will show up in the new plot. For example, let's consider the leftmost point in the original plot of $x(t)$. Note that it occurs when the argument of $x(t)$ is a. So, in the plot of $x\,(mt + c)$, it will occur at t such that $mt + c = a$. Therefore, it will be at $t = \frac{a-c}{m}$.

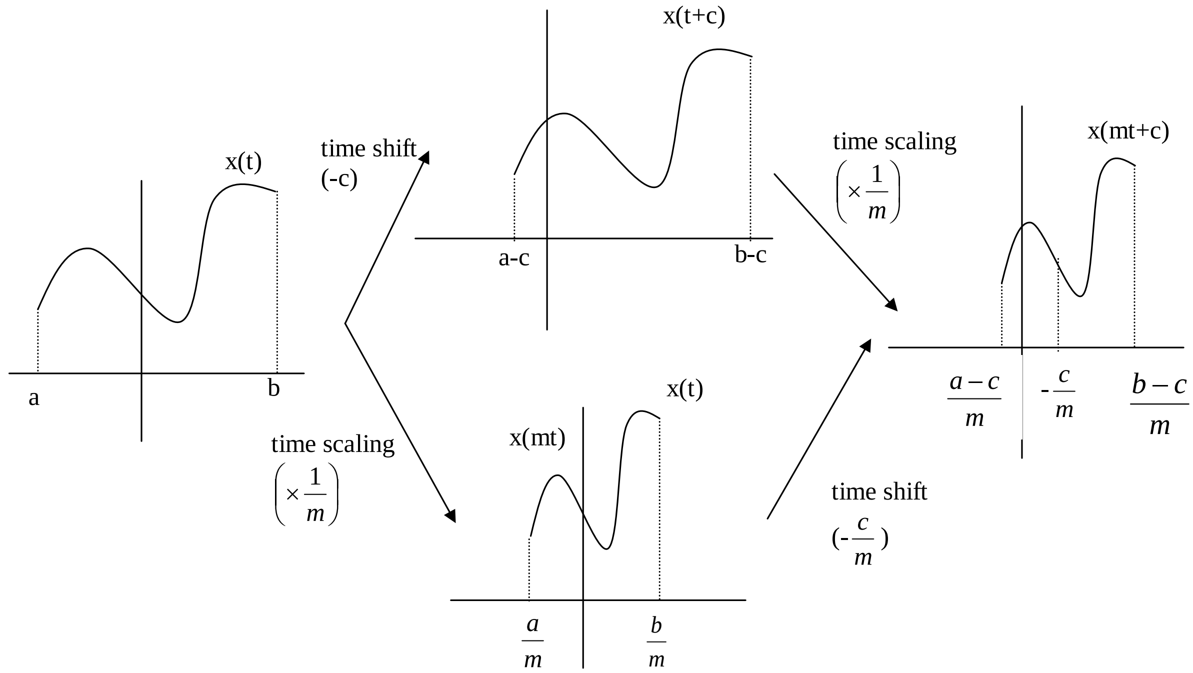


Figure 89: Two approaches for drawing $x\,(mt + c)$.

Example B.3. Consider the function $x(t)$ given in Figure 90.

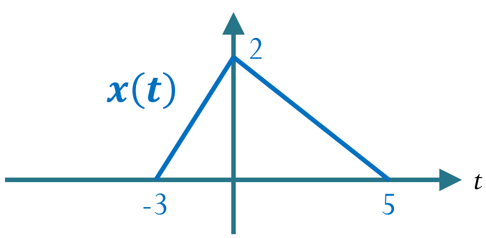


Figure 90: The function $x(t)$ used in Example B.3.

(a) Find the area under $x(t)$.
Solution: See the top part of Figure 91.

(b) Plot and find the area under the new function $x(-t)$.
Solution: See Figure 91.

(c) Plot and find the area under $x(2t)$.
Solution: See Figure 92 for derivation of the plot then see Figure 93 for calculation of the area.

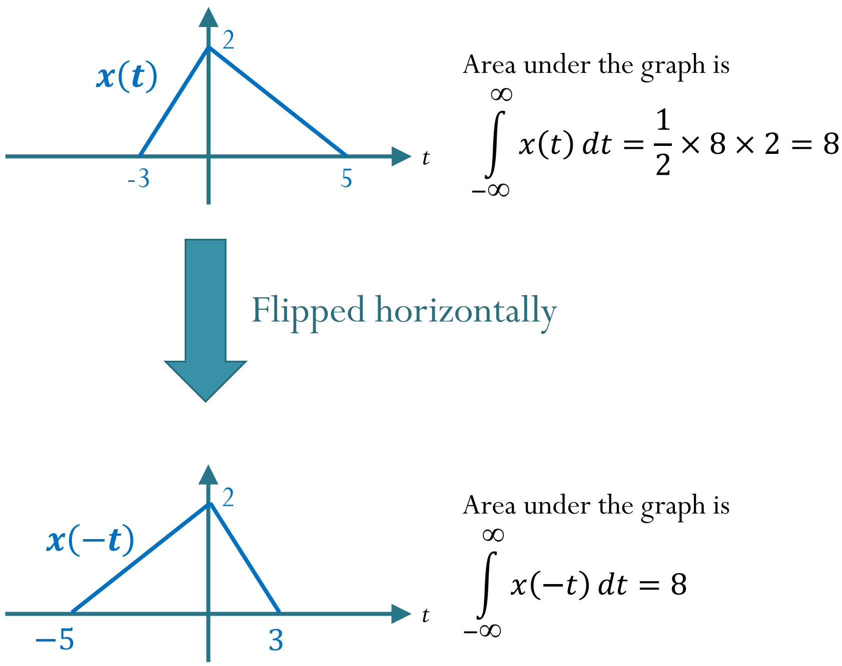


Figure 91: Example of time inversion.

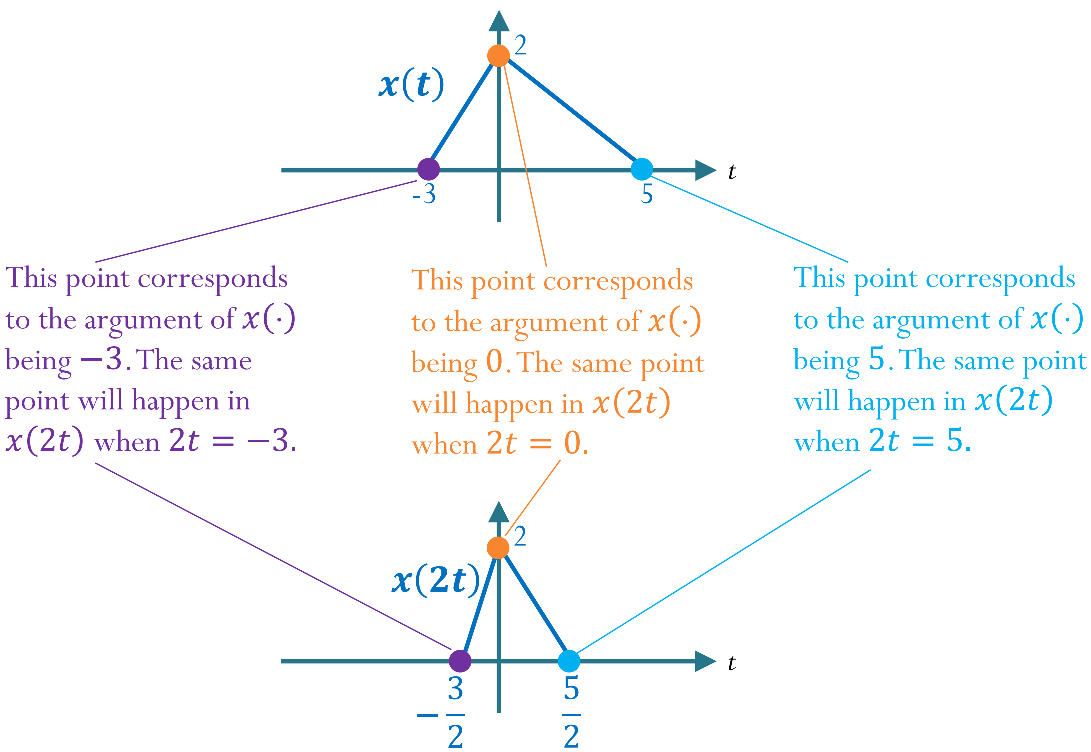


Figure 92: Example of time scaling.

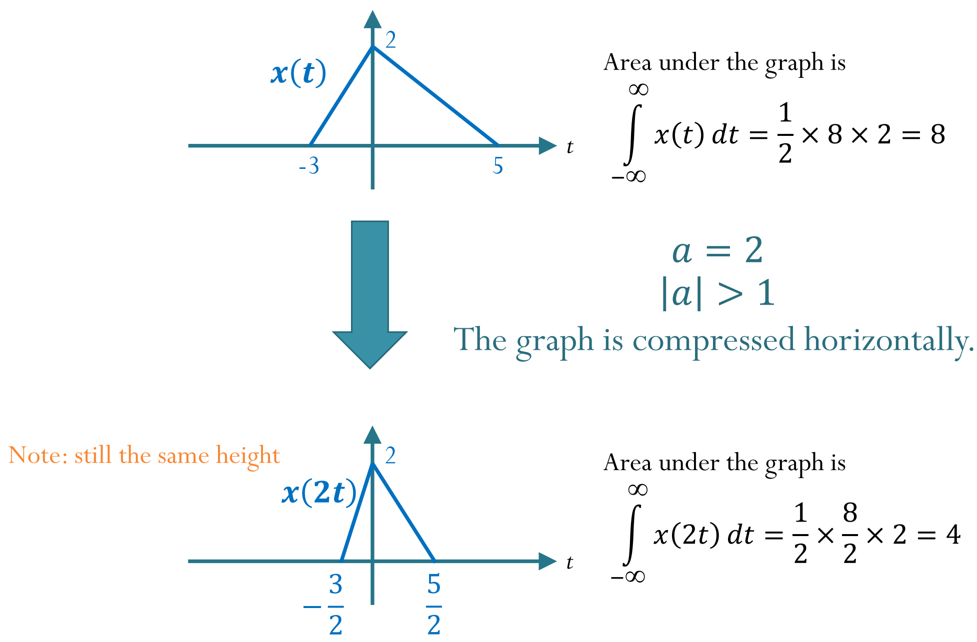


Figure 93: Calculating the area when the function is scaled horizontally.

B.4. Figure 94 shows the delta functions as limits of rectangular functions whose width are compressed to 0 while the areas are kept constant.

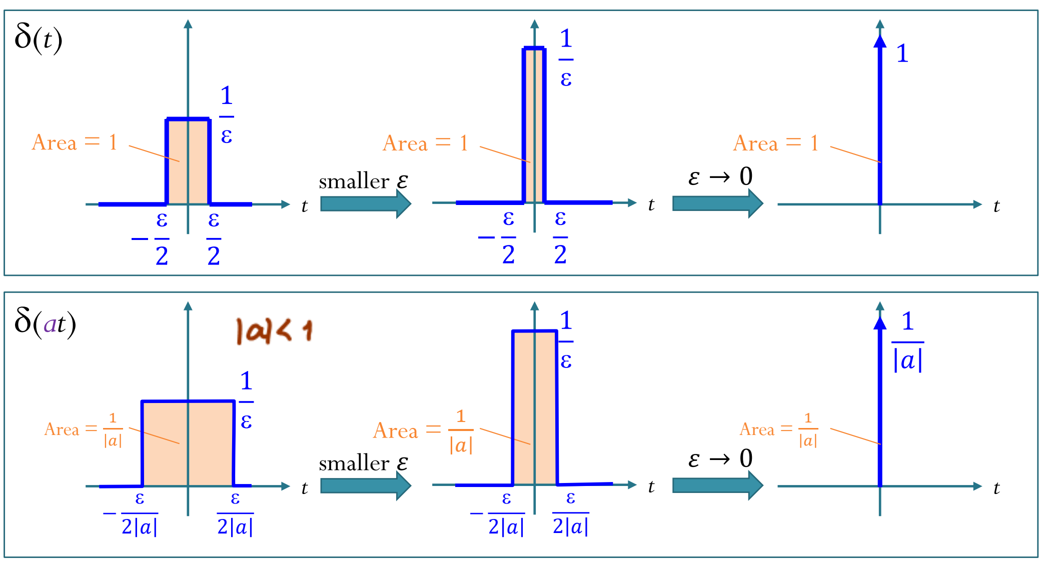


Figure 94: Delta functions as limits of rectangular functions.Here, $a = \frac{1}{2}$.